Working out exam 19th januari 2009 State Space Control 5SC20

① a
$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -g + \frac{c}{m}\frac{u}{x_1} \end{cases}$$

b $g(t_f) = \begin{pmatrix} x_1(t_f) - h_f \\ x_2(t_f) \end{pmatrix}$

c $H = \frac{1}{2}(x_1 - h_f)^2 + \frac{1}{2} r u^2 + \lambda_1 x_2 + \lambda_2 \left(-g + \frac{c}{m}\frac{u}{x_1}\right)$

d $\frac{\partial H}{\partial u} = 0 \Rightarrow r u + \lambda_2 \frac{c}{m}\frac{1}{x_1} \Rightarrow u = -\frac{c}{mr}\frac{\lambda_2}{x_1}$

$$\begin{cases} \dot{\lambda}_1 = -\frac{\partial H}{\partial x_1} = -x_1 + h_f + \frac{c}{m}\frac{\lambda_2 u}{x_1^2} \\ \dot{\lambda}_2 = -\frac{\partial H}{\partial x_2} = -\lambda_1 \end{cases}$$

e $f = \begin{pmatrix} x_2 \\ -g + \frac{c}{m}\frac{u}{x_1} \end{pmatrix}$ $\quad s = -\frac{c}{mr}\frac{\lambda_2}{x_1}$ $\quad h = \begin{pmatrix} -x_1 + h_f - \frac{c^2}{m^2 r}\frac{\lambda_2^2}{x_1^3} \\ -\lambda_1 \end{pmatrix}$

f IV: $\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} h_0 \\ 0 \end{pmatrix}$ V: $\begin{pmatrix} x_1(t_f) \\ x_2(t_f) \end{pmatrix} = \begin{pmatrix} h_f \\ 0 \end{pmatrix}$ VII: $\begin{pmatrix} \lambda_1(t_f) \\ \lambda_2(t_f) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$

VIII: $H(t_f) = 0$

g $\frac{\partial H}{\partial t} = 0 \Rightarrow \forall t: H(t) = H(t_f) = 0$

h IV initial condition on x   V final condition on x

extra constraint VIII for solution of t_f. Just sufficient constraints to solve for $\lambda(0)$ in theory.

i The optimal controller will not stabilise the plant; this is not necessary for a limited time window as applied.

② a at t_f: $\dot{x} = 0 \Rightarrow x_2(t_f) = 0$ $\quad -g + \frac{c}{m}\frac{u}{x_1} = 0 \Rightarrow u_f = \frac{mg}{c} h_f$

b $d\dot{x} = \frac{\partial f}{\partial x^T}\Big|_{t_f} dx + \frac{\partial f}{\partial u^T}\Big|_{t_f} du \Rightarrow d\dot{x}_1 = dx_2$

$d\dot{x}_2 = -\frac{c}{m}\frac{u}{x_1^2}\Big|_{t_f} dx_1 + \frac{c}{m}\frac{1}{x_1}\Big|_{t_f} du = -\frac{c}{m}\frac{u_f}{h_f^2} dx_1 + \frac{c}{m}\frac{1}{h_f} du$

c $a = -\frac{\cancel{c}}{\cancel{m}}\frac{\cancel{m}g}{\cancel{c}}\cancel{h_f}\frac{1}{h_f^{\cancel{2}}} = \frac{-g}{h_f}$ $\qquad b = \frac{c}{m}\frac{1}{h_f}$

d $\det(sI - A) = s^2 + a \Rightarrow s_{1,2} = \pm j\sqrt{|a|} = \pm j\sqrt{\frac{g}{h_f}}$

③ a obviously $a = -1$